The Scheduling Job-Set Optimization Problem: A Model-based Diagnosis Approach

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Motivation

- Scheduling of jobs crucial in production industry
  - Industry 4.0
  - Make to order production / Lean manufacturing
  - (Jobs can be thought of products to be produced)
- Types of job scheduling problem depends on factory layout and products:
  - (Flexible) Job shop scheduling problem
  - Flow shop scheduling problem
  - Open shop scheduling problem
  - Parallel machine scheduling problem
  - ...
- Common challenge in industry: too many jobs than can be accomplished by a factory within planning horizon
  - In other words: not possible to produce all products until a predefined deadline (=planning horizon)
- Problem: Which of the jobs should be cancelled / postponed?
- Set of cancelled / postponed jobs should be optimal
  - What means *optimal*?

Optimality of set of cancelled jobs

1. A set of cancelled jobs should be at least Pareto efficient
   - Subset minimality is a form of Pareto efficiency
2. Depending on application scenario further optimization criteria could be important, e.g.:
   - Revenue loss
   - Customer priorities
   - Marketing strategies

- Associated computational problems:
  1. Job Maximization Problem (JMP)
     - guarantee Pareto efficiency (=subset minimality) of set of cancelled jobs
  2. Job Optimization Problem (JOP)
     - Each job has a defined utility (based on e.g. revenue)
     - Sum of utilities of cancelled jobs should be minimal (= utilities of remaining jobs should be maximal)
  - JMP is (NP-)easy
     - A first minimal diagnosis can be calculated in linear time
  - JOP is NP-hard
     - Finding the best minimal diagnosis may need the calculation of all minimal diagnoses in worst case
  - Note: complexity of theory checking depends on the type of job scheduling problem
     - Often NP-hard

Example for job scheduling problem: job shop scheduling problem (JSSP)

- Strongly NP-hard
- Jobs consist of a predefined sequence of operations (=production steps)
  - Each operation can be performed by a predefined machine
  - Operations have predefined durations
  - A succeding operation can only start after the preceeding operation has been finished
- Machines can perform operations one by one
  - Non-preemption
- Cost of a schedule = completion time (= timespan to perform all operations)

Example cont.: Problem instance and optimal solution

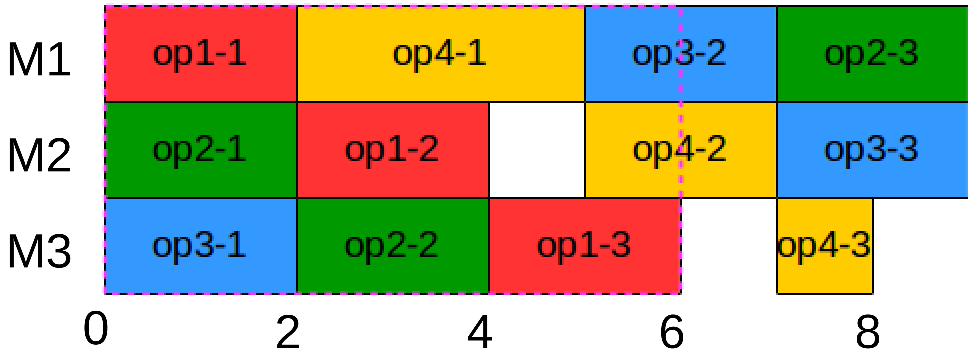
```
Job 1:  op1-1 (type=machine1 | length=2)
        op1-2 (type=machine2 | length=2)
        op1-3 (type=machine3 | length=2)

Job 2:  op2-1 (type=machine2 | length=2)
        op2-2 (type=machine3 | length=2)
        op2-3 (type=machine1 | length=2)

Job 3:  op3-1 (type=machine3 | length=2)
        op3-2 (type=machine1 | length=2)
        op3-3 (type=machine2 | length=2)

Job 4:  op4-1 (type=machine1 | length=3)
        op4-2 (type=machine2 | length=2)
        op4-3 (type=machine3 | length=1)
```

- Optimal solution with completion time = 9
- What if deadline is at 6?



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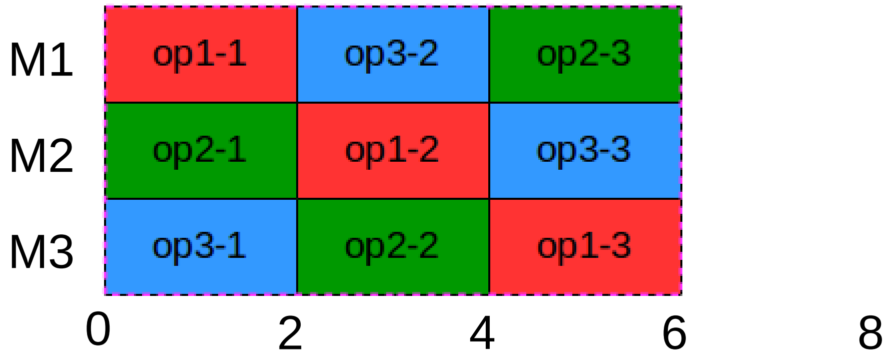
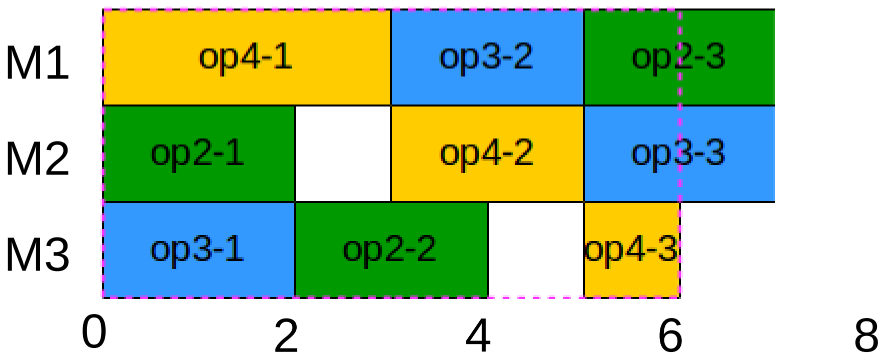
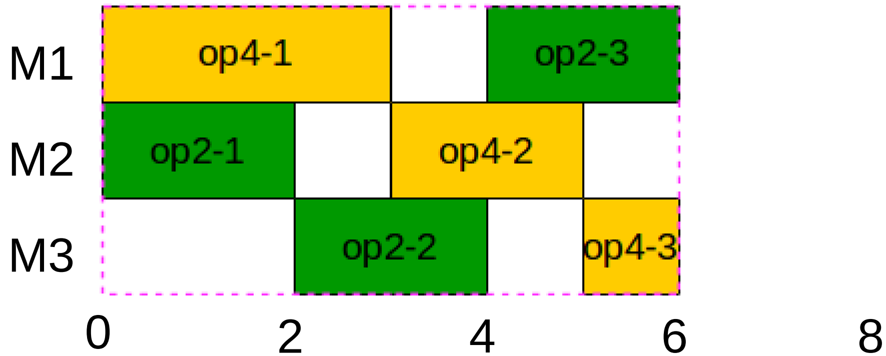
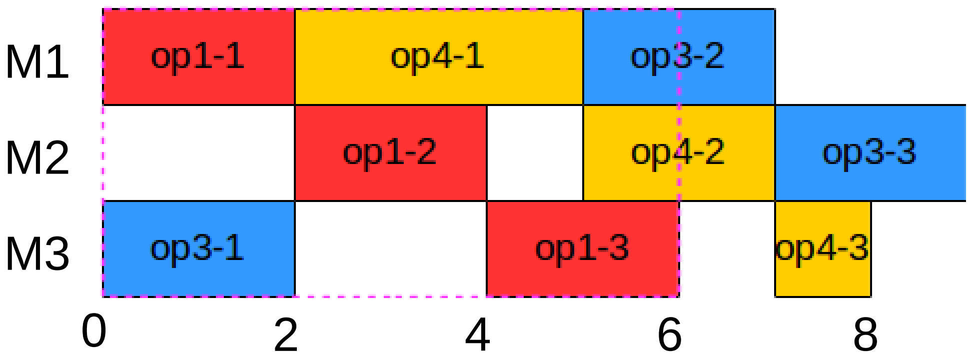
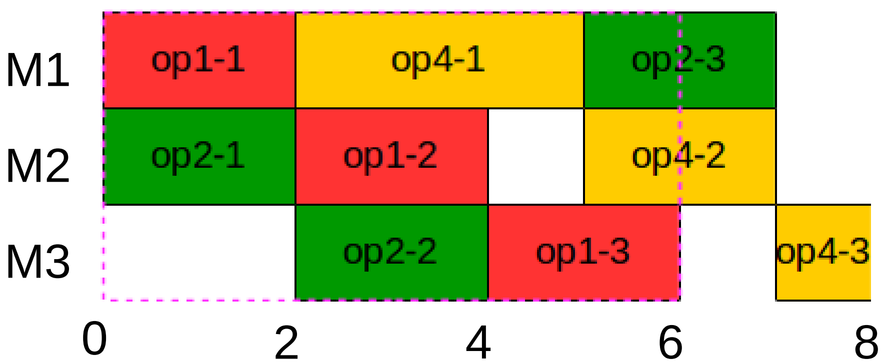
Example cont.: JMP/JOP











- Two solutions for **JMP** (=minimal diagnoses):
  - ✔ cancel job 4, or
  - ✔ cancel job 1 and job 3
- **JOP**:
  - If job utilities are equal:
    - ➔ Minimum cardinality diagnosis
    - ✓ cancel job 4
  - If job utilities are: $u_1=2$, $u_2=3$, $u_3=1$, $u_4=4$
    - ➔ $u_1 + u_3 < u_4$
    - ✓ cancel job 1 and job 3

Evaluation

- Basic question addressed: Can model-based diagnosis algorithms be applied on JMP/JOP?
  - Proof of concept
- Job shop scheduling problem
  - Taillard benchmark
    - 50 jobs / 15 machines (each job consists of 15 ops)
    - 100 jobs / 20 machines (each job consists of 20 ops)
    - 10 problem instances for each class (i.e. 20 in total)
  - optimal completion times (κ^*) are known
  - deadlines are set to $\kappa = r \times \kappa^*$, $r \in \{0.95, 0.9, 0.85, 0.8, 0.75\}$
- Reverse QuickXplain for diagnosis calculation
- IBM CP Optimizer for scheduling (=consistency checking)

Results

r	(50 jobs, 15 machines) diag size	time	(100 jobs, 20 machines) diag size	time
0.95	3	137	4	407
0.95	3	169	4	152
0.95	4	99	4	446
0.95	2	93	5	230
0.95	2	302	4	842
0.95	3	97	6	287
0.95	3	22	5	321
0.95	3	27	5	230
0.95	3	185	4	200
0.95	3	207	5	333
0.9	7	91	10	392
0.9	5	250	10	209
0.9	6	223	9	965
0.9	5	42	10	401
0.9	5	425	9	803
0.9	6	36	11	166
0.9	6	34	10	391
0.9	5	107	10	375
0.9	5	375	9	409
0.9	5	453	9	834
0.85	9	42	15	358
0.85	8	651	18	264
0.85	9	664	14	1092
0.85	8	30	15	313
0.85	7	657	14	991
0.85	10	34	17	173
0.85	7	178	15	476
0.85	8	106	15	217
0.85	7	5250	15	230
0.85	8	68	15	299

r	(50 jobs, 15 machines) diag size	time	(100 jobs, 20 machines) diag size	time
0.8	13	28	20	490
0.8	11	571	24	654
0.8	—	—	21	759
0.8	10	99	21	266
0.8	10	450	20	790
0.8	11	42	21	344
0.8	11	159	19	397
0.8	10	613	19	688
0.8	11	3093	20	291
0.8	12	140	20	350
0.75	13	79	25	956
0.75	14	5497	30	446
0.75	—	—	25	2168
0.75	—	—	28	308
0.75	—	—	25	1830
0.75	13	125	28	350
0.75	14	66	24	943
0.75	13	607	25	389
0.75	—	—	24	534
0.75	14	695	26	434

Solutions for:

- all (100,20) instances with all $r \in \{0.95, 0.9, 0.85, 0.8, 0.75\}$
- all (50,15) instances with $r \in \{0.95, 0.9, 0.85\}$
- 90 % of the (50,15) instances with r = 0.8
- 60 % of the (50,15) instances with r = 0.75

Conclusions

- Model-based diagnosis can be applied to JMP also for industrial size instances
- JMP solutions seem to be near optimal solutions for JOP
  - Further heuristic methods can easily be invented for JOP based on JMP
- Future work:
  - Other (heuristic) diagnosis algorithms
  - Application to job scheduling problems other than JSSP
  - Tuning of scheduling machine (= consistency checker)
  - …
- Thank you for your attention!